

# 35 Lecture - CS301

## Important Subjective

### 1. What is the dynamic equivalence problem, and what are its practical applications?

Answer: The dynamic equivalence problem is the problem of efficiently maintaining equivalence relations under dynamic changes to a set of elements. It arises in many areas of computer science, such as databases, information retrieval, and natural language processing.

### 2. How does the disjoint-set data structure solve the dynamic equivalence problem?

Answer: The disjoint-set data structure uses a set of trees to represent the equivalence classes of elements, with each tree rooted at the representative element of its equivalence class. The find and union operations are used to determine the equivalence class of an element and combine two equivalence classes into a single class, respectively.

### 3. What is path compression, and how does it improve the performance of the disjoint-set data structure?

Answer: Path compression is a modification to the disjoint-set data structure that involves setting the parent of each node in the path from a node to its representative element to the representative element itself. This improves the time complexity of the find operation by reducing the height of the trees representing the equivalence classes.

### 4. What is rank-based union, and how does it improve the performance of the disjoint-set data structure?

Answer: Rank-based union is a modification to the disjoint-set data structure that involves attaching the smaller tree to the root of the larger tree during the union operation. This improves the time complexity of the union operation by reducing the height of the trees representing the equivalence classes.

### 5. What is the time complexity of the find and union operations in the disjoint-set data structure, and how do they depend on the size of the input?

Answer: The time complexity of the find and union operations in the disjoint-set data structure is  $O(\log n)$ , where  $n$  is the size of the input. This is because the height of the trees representing the equivalence classes is bounded by  $\log n$ .

### 6. What is the worst-case time complexity of the find and union operations in the disjoint-set data structure, and when does it occur?

Answer: The worst-case time complexity of the find and union operations in the disjoint-set data structure is  $O(n)$ , which occurs when the tree representing the equivalence classes is a linear chain.

**7. What modifications to the standard disjoint-set data structure can be used to improve its performance?**

Answer: Path compression, rank-based union, and weighted union are modifications to the standard disjoint-set data structure that can improve its performance.

**8. How does the weighted union technique improve the performance of the disjoint-set data structure?**

Answer: Weighted union is a modification to the disjoint-set data structure that involves attaching the smaller tree to the root of the larger tree during the union operation, similar to rank-based union. However, the size of each tree is tracked, and the root of the smaller tree is attached to the root of the larger tree to minimize the increase in the height of the tree.

**9. What is the transitive closure of a relation, and how is it related to the dynamic equivalence problem?**

Answer: The transitive closure of a relation is the smallest transitive relation that contains the original relation. It is related to the dynamic equivalence problem because finding the transitive closure of a relation involves determining the equivalence classes of elements.

**10. How does the dynamic equivalence problem relate to the concept of clustering in machine learning?**

Answer: The dynamic equivalence problem is closely related to the concept of clustering in machine learning, as both involve grouping similar elements into equivalence classes. However, clustering is a more general problem that does not necessarily involve maintaining the equivalence classes under dynamic changes to the input.